Large-scale Monte Carlo simulations of the three-dimensional *XY* spin glass

J. H. Pixley and A. P. Young*
Department of Physics, University of California, Santa Cruz, California 95064, USA
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We study the XY spin glass by large-scale Monte Carlo simulations for sizes up to 24^3, down to temperatures below the transition temperature found in earlier work. The data for the larger sizes show more marginal behavior than that for the smaller sizes, indicating that the lower critical dimension is close, and possibly equal to three. We find that the spins and chiralities behave in a similar manner. We also address the optimal ratio of "over-relaxation" to "Metropolis" sweeps in the simulation.

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I. INTRODUCTION

Following the convincing numerical work of Ballesteros *et al.*,[1] there has been little doubt that Ising spin glasses in three dimensions have a finite-temperature transition. In this paper we shall study a related model for which the existence of a finite-temperature transition is more controversial: the isotropic XY spin glass, which is composed of classical spins with two components. Early work on this model in three dimensions[2,3] indicated a zero-temperature transition, or possibly a transition at a very low but nonzero temperature. However, following the pioneering work of Villain,[4] which emphasized the role of "chiralities" (Ising-like variables which describe the handedness of the noncollinear spin structures), Kawamura and Tanemura[5] proposed that the spin-glass transition only occurs at $T_{SG}=0$ and that a *chiral*-glass transition occurs at a finite temperature T_{CG}. This scenario requires that spins and chiralities *decouple* at long length scales. Kawamura and collaborators have given numerical results in favor of this scenario.[6]

However, the absence of a spin-glass transition in the XY spin glass has been challenged by Maucourt and Grempel[7] and subsequently Akino and Kosterlitz,[8] who found evidence for a possible finite T_{SG} from zero-temperature domain wall calculations. Furthermore, by studying the dynamics of the XY spin glass in the phase representation, Granato[9] found that the "current-voltage" characteristics exhibited scaling behavior, which he interpreted as a transition in the spins, as well as the chiralities.

In earlier work,[10] referred to as LY, Lee and one of the present authors studied spin and chiral correlations on an equal footing, using the method of analysis that was the most successful for the Ising spin glass,[1,11,12] namely finite-size scaling of the correlation length. Considering a modest range of sizes, $N=L^3$ with $L\leq 12$, LY found that the behavior of spins and chiralities was quite similar, and they both had a finite-temperature transition, apparently at the same temperature.

LY studied both XY and Heisenberg models, finding similar conclusions for both. However, for the Heisenberg case, subsequent studies on much larger sizes,[13,14] up to $L=32$, have painted a more complex picture. The data at the lowest temperatures and largest sizes seem rather "marginal," i.e., the system is close to the lower critical dimension, where the finite-temperature phase transition is removed by fluctuations. The data for spins and chiralities are still quite similar, although not identical, and do not seem to give compelling evidence for spin-chirality decoupling as proposed by Kawamura. In addition, Hukushima and Kawamura[15] have also studied somewhat larger sizes than LY ($L\leq 20$), but they argued that their data *are* consistent with spin-chirality decoupling.

It is of interest to know whether the "crossover" to more marginal behavior found for larger sizes is special to the three-component case, or whether the same situation occurs quite generally with vector spin glasses. In this paper, we therefore study the XY (two-component) spin glass for larger sizes (up to 24^3) than in LY (which went only up to 12^3). We find a situation that is quite similar to the Heisenberg case, namely marginal behavior for low T and large sizes. The behaviors of the spin-glass and chiral-glass correlation length are *very* similar, more similar than was the case for the Heisenberg spin glass, and does not appear to provide evidence for spin-chirality decoupling, at least up to the sizes studied.

Simulations on very large sizes for vector spin glasses have been possible because including "over-relaxation" moves, in addition to the more familiar Metropolis or heat bath moves, speeds up equilibration.[16] A second motivation of the present work is to investigate quantitatively the *optimal* ratio of over-relaxation to Metropolis sweeps for the XY spin glass.

The layout of this paper is as follows. Sec. II describes the model, the parameters of the simulations, and the finite-size scaling approach. The results for the correlation length are presented in Sec. III. In Sec. IV we estimate the optimal ratio between the number of over-relaxation and Metropolis sweeps, and Sec. V summarizes our conclusions.

II. MODEL AND ANALYSIS

We use the standard Edwards-Anderson XY spin-glass model

$$\mathcal{H}=-\sum_{\langle i,j\rangle} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j, \tag{1}$$

where $\mathbf{S}_i$ are two-component classical vectors of unit length at the sites of a simple cubic lattice, and J_{ij} are nearest-neighbor interactions with a Gaussian distribution with zero mean and standard deviation unity. Periodic boundary condi-

tions are applied on lattices with $N=L^3$ spins.

The spin-glass order parameter, $q^{\mu\nu}(\mathbf{k})$, at wave vector $\mathbf{k}$, is defined to be

$$q^{\mu\nu}(\mathbf{k}) = \frac{1}{N}\sum_i S_i^{\mu(1)} S_i^{\nu(2)} e^{i\mathbf{k}\cdot\mathbf{R}_i}, \tag{2}$$

where μ and ν are spin components, and "(1)" and "(2)" denote two identical copies of the system with the same interactions. From this we determine the wave-vector-dependent spin-glass susceptibility $\chi_{\mathrm{SG}}(\mathbf{k})$ by

$$\chi_{\mathrm{SG}}(\mathbf{k}) = N\sum_{\mu,\nu} [\langle |q^{\mu\nu}(\mathbf{k})|^2\rangle]_{\mathrm{av}}, \tag{3}$$

where $\langle\cdots\rangle$ denotes a thermal average and $[\cdots]_{\mathrm{av}}$ denotes an average over disorder. The spin-glass correlation length is then determined[1,17] from

$$\xi_L = \frac{1}{2\,\sin(k_{\mathrm{min}}/2)}\left[\frac{\chi_{\mathrm{SG}}(0)}{\chi_{\mathrm{SG}}(\mathbf{k}_{\mathrm{min}})} - 1\right]^{1/2}, \tag{4}$$

where $\mathbf{k}_{\mathrm{min}}=(2\pi/L)(1,0,0)$.

For the XY spin glass, chirality of a square is[6]

$$\kappa_i^{\mu} = \frac{1}{2\sqrt{2}}{\sum_{l,m}}' \mathrm{sgn}(J_{lm})\sin(\theta_l - \theta_m), \tag{5}$$

where θ_l is the angle characterizing the direction of spin $\mathbf{S}_l$, and the prime on the sum indicates that it is over the four bonds around the elementary plaquette perpendicular to the μ axis whose "bottom left" corner is site i. The chiral-glass susceptibility is then given by

$$\chi_{\mathrm{CG}}^{\mu}(\mathbf{k}) = N[\langle |q_c^{\mu}(\mathbf{k})|^2\rangle]_{\mathrm{av}}, \tag{6}$$

where the chiral overlap $q_c^{\mu}(\mathbf{k})$ is given by

$$q_c^{\mu}(\mathbf{k}) = \frac{1}{N}\sum_i \kappa_i^{\mu(1)}\kappa_i^{\mu(2)} e^{i\mathbf{k}\cdot\mathbf{R}_i}. \tag{7}$$

We define the chiral correlation lengths $\xi_{c,L}^{\mu}$ by

$$\xi_{c,L}^{\mu} = \frac{1}{2\,\sin(k_{\mathrm{min}}/2)}\left[\frac{\chi_{\mathrm{CG}}(0)}{\chi_{\mathrm{CG}}^{\mu}(\mathbf{k}_{\mathrm{min}})} - 1\right]^{1/2}, \tag{8}$$

in which $\chi_{\mathrm{CG}}(\mathbf{k}=0)$ is independent of μ. Note that $\xi_{c,L}^{\mu}$ will, in general, be different for $\hat{\mu}$ along $\mathbf{k}_{\mathrm{min}}$ (the $\hat{x}$ direction) and perpendicular to $\mathbf{k}$, although this difference is very small for large sizes. The results presented will be an average over the three (two transverse and one longitudinal) correlation lengths.

To equilibrate the system efficiently we perform three types of Monte Carlo move. First we use "over-relaxation" sweeps[16] in which we sweep sequentially through the lattice, and, at each site, compute the local field on the spin, $\mathbf{H}_i = \Sigma_j J_{ij}\mathbf{S}_j$. The new value for the spin on site i is taken to be its old value reflected about $\mathbf{H}$, i.e.,

$$\mathbf{S}_i' = -\mathbf{S}_i + 2\frac{\mathbf{S}_i\cdot\mathbf{H}_i}{H_i^2}\mathbf{H}_i. \tag{9}$$

Over-relaxation sweeps preserve energy and so are also known as microcanonical sweeps.

Second, we include Metropolis sweeps since, unlike the over-relaxation sweeps, these *do* change the energy, and so are needed to bring the system to equilibrium. For the data presented in Secs. II and III, we do one Metropolis sweep after every ten over-relaxation sweeps. As for the over-relaxation case, we sweep sequentially through the lattice. To update a given spin, we choose a trial new direction randomly within a window $\pm\Delta\theta/2$ of the current direction, and accept this new direction with the usual Metropolis probability, $\min[1,\exp(-\beta\Delta E)]$, where $\beta=1/T$ and ΔE is the energy difference between the trial state and the current state. We choose the window size $\Delta\theta$ to vary with temperature in such a way that the acceptance ratio for Metropolis moves is in the range of 30 to 50%.

A Metropolis sweep requires more CPU time than an over-relaxation sweep, so we do mainly over-relaxation sweeps, including *some* Metropolis sweeps only to change the energy from time to time to ensure that the algorithm is ergodic. In fact, as discussed in Sec. IV, including a fraction of over-relaxation sweeps not only *reduces the CPU time* (for a given *total* number of sweeps) but also *reduces the number of sweeps* needed to equilibrate.

Finally we do "parallel tempering" sweeps,[18,19] which are necessary to prevent the system being trapped in a valley in configuration space at low temperatures. One takes N_T copies of the system with the same bonds but at a range of different temperatures. The minimum temperature, $T_{\mathrm{min}}\equiv T_1$, is the low temperature where one wants to investigate the system (below T_{SG} in our case), and the maximum, $T_{\mathrm{max}}\equiv T_{N_T}$, is high enough that the system equilibrates very fast (well above T_{SG} in our case). A parallel tempering sweep consists of swapping the temperatures of the spin configurations at a pair of neighboring temperatures, T_i and T_{i+1}, for $i=1,2,\cdots,T_{N_T-1}$ with a probability that satisfies the detailed balance condition. Further details on the application to vector spin glasses can be found in Ref. 14. For the simulations in Secs. II and III we do one parallel tempering sweep after each Metropolis sweep. Table I gives the parameters of the simulations used to collect the data in Secs. II and III.

To test for equilibration[20] we require that data satisfy the relation[14]

$$U = U(q_l, q_s), \tag{10}$$

where

$$U(q_l, q_s) = \frac{z}{2T}(q_l - q_s), \tag{11}$$

which is valid for a Gaussian bond distribution. Here $U=-[\Sigma_{\langle i,j\rangle}J_{ij}\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle]_{\mathrm{av}}$ is the average energy per spin, $q_l=(1/N_b)\Sigma_{\langle i,j\rangle}[\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle^2]_{\mathrm{av}}$ is the "link overlap," $q_s=(1/N_b)\Sigma_{\langle i,j\rangle}[\langle(\mathbf{S}_i\cdot\mathbf{S}_j)^2\rangle]_{\mathrm{av}}$, $N_b=(z/2)N$ is the number of nearest-neighbor bonds, and z (=6 here) is the lattice coordination number. Equation (10) is easily derived by integrating by parts the expression for the average energy with respect to J_{ij}, noting that the average $[\cdots]_{\mathrm{av}}$ is over a Gaussian function of the J_{ij}'s.

TABLE I. Parameters of the simulations described in Secs. II and III. N_{samp} is the number of samples, $N_{\text{equil}}^{\text{OR}}$ is the number of over-relaxation Monte Carlo sweeps for equilibration for each of the $2N_T$ replicas for a single sample, and $N_{\text{meas}}^{\text{OR}}$ is the number of over-relaxation sweeps for measurement. The number of Metropolis sweeps and the number of parallel tempering sweeps are both equal to 10% of the number of over-relaxation sweeps. T_{min} and T_{max} are the lowest and highest temperatures simulated, and N_T is the number of temperatures used in the parallel tempering.

L	N_{samp}	$N_{\text{equil}}^{\text{OR}}$	$N_{\text{meas}}^{\text{OR}}$	T_{min}	T_{max}	N_T
4	5000	1280	1280	0.200	1.40	11
6	5001	10240	10240	0.200	1.40	19
8	1000	40960	40960	0.200	1.40	27
12	1000	81920	81920	0.250	0.60	24
16	1006	409600	409600	0.265	0.60	32
24	461	2457600	2457600	0.265	0.45	35

The spins are initialized in random directions so the energy, the left-hand side (LHS) of Eq. (10), is initially close to zero and decreases, presumably monotonically, to its equilibrium value as the length of the simulation increases. Hence, the LHS of Eq. (10) will be too *large* if the simulation is too short to equilibrate the system. On the other hand, the right-hand side (RHS) of Eq. (10),will be too *small* if the simulation is too short because q_l starts off close to zero and then increases with MC time as the two replicas start to find the same local minima. The quantity q_s will be less dependent on Monte Carlo time than q_l since it is a local variable for a single replica. (For the Ising case it is just a constant.) Hence, if the simulation is too short the RHS of Eq. (10) will be too low. In other words, the two sides of Eq. (10) are expected to approach the common equilibrium value from *opposite directions* as the length of the simulation increases. Only if Eq. (10) is satisfied within small error bars do we accept the results of a simulation.

Figure 1 shows a test to verify that Eq. (10) is satisfied at long times. For the parameters used, $L=16, T=0.265$, this occurs when the total number of (over-relaxation) sweeps ($N_{\text{sweep}}^{\text{OR}}=N_{\text{equil}}^{\text{OR}}+N_{\text{meas}}^{\text{OR}}$) is about 2×10^5. Figure 2 shows that the spin and chiral correlation lengths appear to become independent of N_{sweep}, and hence are presumably equilibrated, when N_{sweep} is larger than this *same* value. Hence, it appears that when Eq. (10) is satisfied to high precision, the data for the correlation lengths are equilibrated.

With the number of sweeps shown in Table I, Eq. (10) was satisfied for all sizes and temperatures. The error bars are made sufficiently small by averaging over a large number of samples.

Since ξ_L/L is dimensionless, it has the finite-size scaling form[1,10,17]

$$\frac{\xi_L}{L} = \tilde{X}[L^{1/\nu}(T - T_{\text{SG}})], \tag{12}$$

where ν is the correlation-length exponent. Note that there is no power of L multiplying the scaling function $\tilde{X}$. By contrast, for the spin-glass susceptibility, $\chi_{\text{SG}} \equiv \chi_{\text{SG}}(\mathbf{k}=0)$, which has dimensions, the finite-size scaling form is

$$\chi_{\text{SG}} = L^{2-\eta_{\text{SG}}}\tilde{K}[L^{1/\nu}(T - T_{\text{SG}})], \tag{13}$$

where η_{SG} is a critical exponent. There is an expression analogous to Eq. (12) for the chiral correlation length, and to Eq. (13) for the chiral-glass susceptibility $\chi_{\text{CG}} \equiv \chi_{\text{CG}}(\mathbf{k}=0)$. For the later case, there is no reason to expect that the exponents η_{SG} and η_{CG} are equal.

From Eq. (12) it follows that the data for ξ_L/L for different sizes come together at $T=T_{\text{SG}}$. In addition, they are also

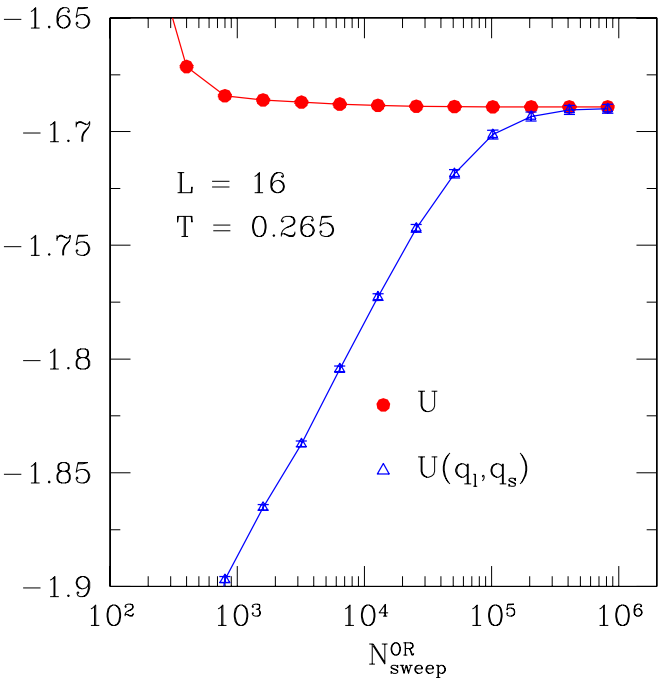

FIG. 1. (Color online) Equilibration plot testing Eq. (10) for $L=16$ at $T=0.265$. It is seen that the data for U and $U(q_l,q_s)$, given by Eq. (11), come together when the total number of over-relaxation sweeps, $N_{\text{sweep}}^{\text{OR}}=N_{\text{equil}}^{\text{OR}}+N_{\text{meas}}^{\text{OR}}$, see Table I, is equal to about 2×10^5. These two quantities then stay at their common value, indicating that equilibration has been achieved. It is seen that the energy comes close to its equilibrium value very quickly, whereas $U(q_l,q_s)$, which depends on the link overlap q_l between two replicas, takes much longer.

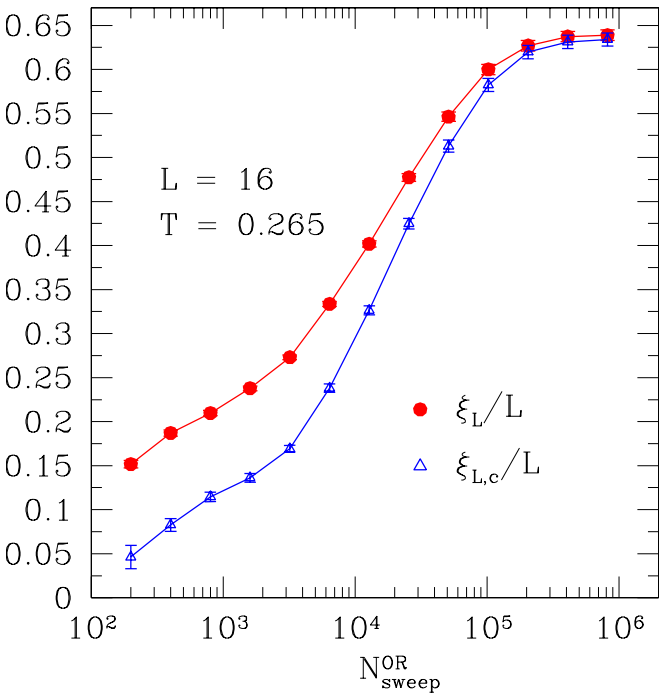

FIG. 2. (Color online) A plot of the spin-glass and chiral-glass correlation lengths, ξ_L and $\xi_{L,c}$, divided by L as a function of the total number of sweeps for $L=16$ at $T=0.265$. It is seen that the data flatten off at around 2×10^5 sweeps, the value where the two sets of data in Fig. 1 start to agree. This indicates that when the data in Fig. 1 agree within high precision, i.e., when Eq. (10) is satisfied, the correlation lengths have reached their equilibrium value.

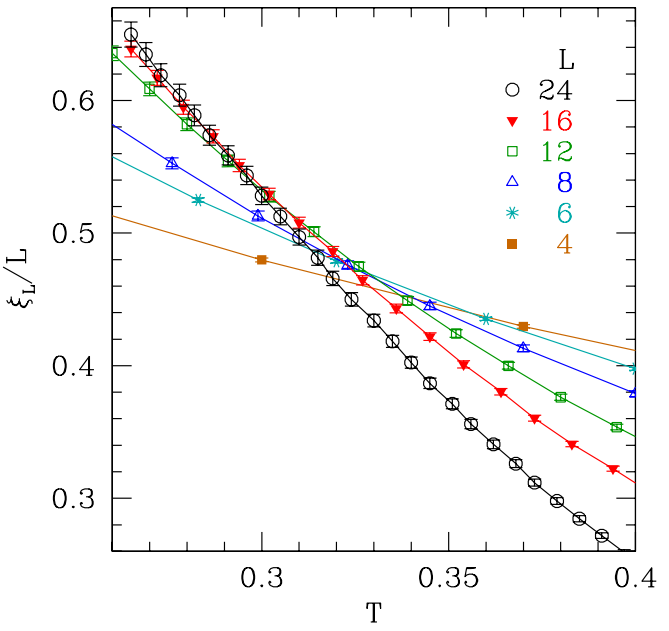

FIG. 3. (Color online) Data for ξ_L/L, the spin-glass correlation length divided by system size as a function of T for different system sizes.

expected to splay out again on the low-T side[1] if there is spin-glass order below T_{SG}. In a marginal situation with a line of critical points, as in the Kosterlitz-Thouless-Berezinskii theory of the transition in the two-dimensional XY ferromagnet, the data for different sizes would come together at T_{SG} and then stick together at lower T, see for example Fig. 3 of Ref. 1.

III. RESULTS

We studied sizes from $L=4$ to $L=24$, as shown in Table I. The CPU time involved to get this data is about 8 Mac G5 CPU years.

The data for the spin-glass correlation length (divided by L) are shown in Fig. 3, and the corresponding data for the chiral-glass correlation length are shown in Fig. 4. In both cases the data for smaller sizes intersect and splay out at lower temperature. However, for the larger sizes the splaying out is small, indicating close to "marginal" behavior, i.e., the "lower critical dimension" is close to three.

The data for the spins and chiralities in Figs. 3 and 4 are very similar, so we do not see evidence for spin-chirality decoupling. To make clearer the similarity between the two sets of data, we plot them both in Fig. 5, including just the three largest sizes. The temperature, where the data merge, decreases slightly with increasing size. We have estimated the temperatures where the data intersect/merge for different pairs of sizes and present the results in Table II. The temperatures are seen to decrease with increasing size. If one neglects the smallest pair of sizes ($L=4/6$), the shift is somewhat bigger for the spins than for the chiralities, but from the data, it is not possible to reliably estimate whether or not the intersection temperature will tend to zero for $L\to\infty$ for either set of data.

In Fig. 6 we present data for the *ratio* of the chiral-glass to spin-glass correlation lengths. For the largest sizes the data intersects for T about 0.33 and then (slightly) splays out in

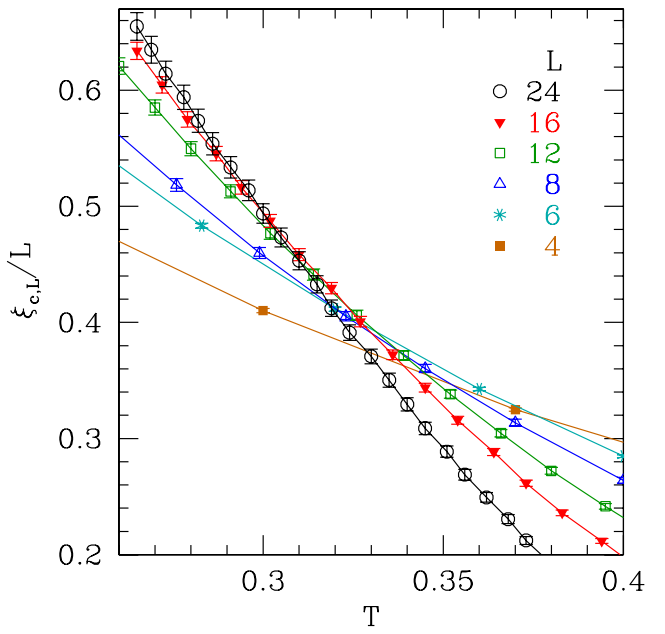

FIG. 4. (Color online) Data for the chiral correlation length (averaged over longitudinal and transverse directions) divided by system size as a function of T for different system sizes.

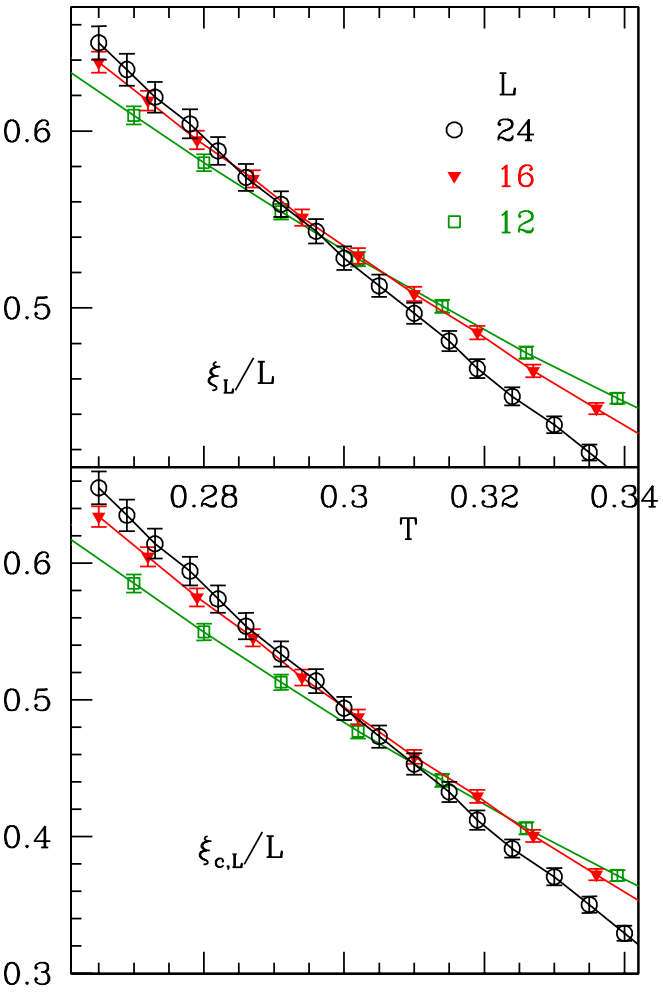

FIG. 5. (Color online) The same data as in Figs. 3 and 4, but including only the largest sizes and in a somewhat expanded scale.

the low-T side. If there is a single transition involving both spins and chiralities, then the data would become independent of size at the transition since both ξ_L and $\xi_{c,L}$ are proportional to L there [see Eq. (12)]. If the stiffness exponents for spins and chiralities are equal (we are not aware of any argument for this even if there is a single transition), then the data would become independent of L for large L at low T. If the stiffness exponent for chiralities is larger than that for the spins, then the ratio would diverge in this limit. From the data it is not possible to say for sure if the data diverge or not

TABLE II. Estimated crossing temperatures for the spin- and chiral-glass correlation lengths. The results are given to the nearest 0.005, but the uncertainties are greater than this because of the error bars in the data itself.

Sizes	T_{crossing} (spins)	T_{crossing} (chiralities)
4/6	0.355	0.375
6/8	0.33	0.32
8/12	0.33	0.335
12/16	0.31	0.32
16/24	0.285	0.30

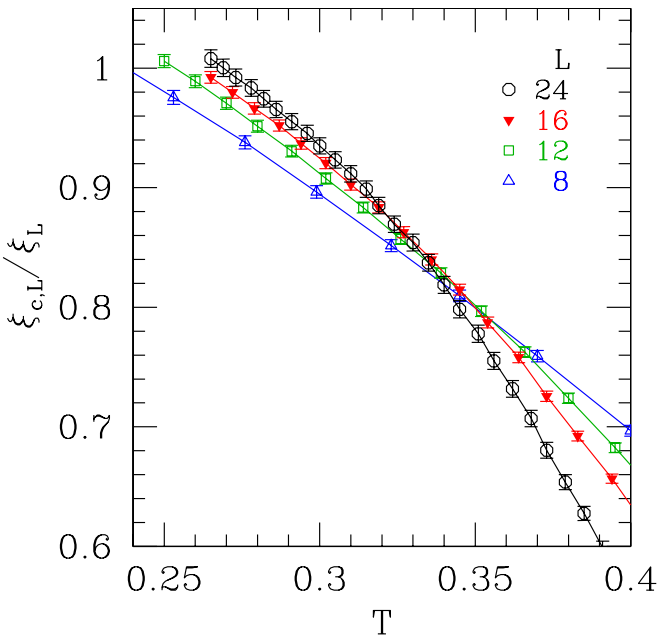

FIG. 6. (Color online) Data for the *ratio* of the chiral-glass to the spin-glass correlation lengths for sizes from 8 to 24.

at low T, but the size dependence at the larger sizes is very weak.

In the spin-chirality decoupling scenario, the ratio would diverge even at the transition, and there would not be a common intersection. We feel that the data of Fig. 6 reinforce our view that if spin-chirality decoupling occurs, one would need even larger sizes than $L=24$ to see it.

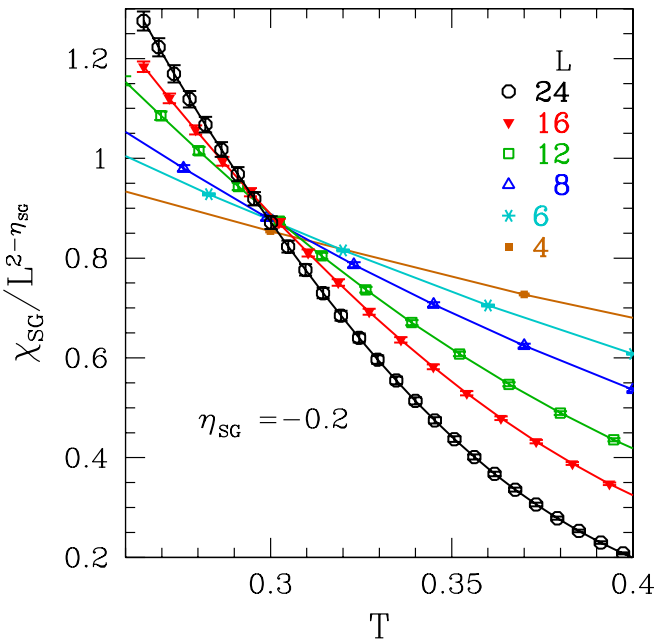

FIG. 7. (Color online) Data for the spin-glass susceptibility $\chi_{SG} \equiv \chi_{SG}(\mathbf{k}=0)$ divided by $L^{2-\eta_{SG}}$, where we took $\eta_{SG}=-0.2$ in order to get the data to intersect [see Eq. (13)], for T around 0.30 since this is roughly where the data for ξ_L/L and $\xi_{c,L}$ intersect/merge for the largest sizes (see Figs. 3 and 4).

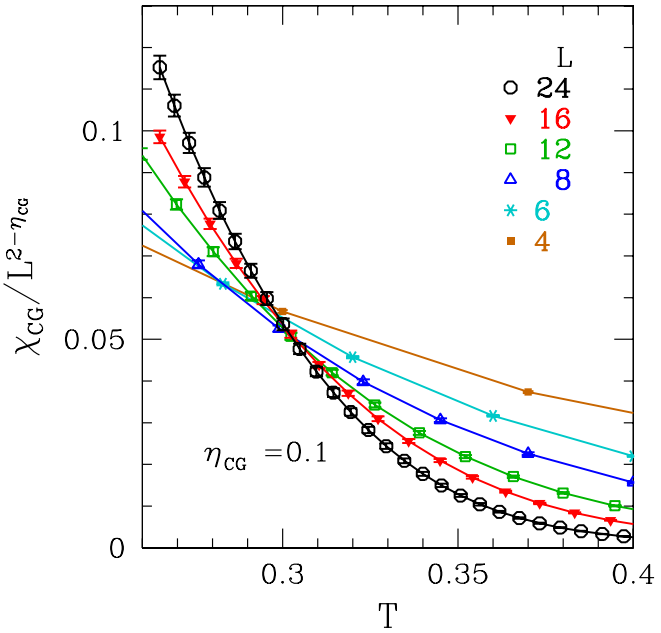

FIG. 8. (Color online) Similar to Fig. 7 but for the chiral-glass susceptibility $\chi_{CG} \equiv \chi_{CG}(\mathbf{k}=0)$. Here we took $\eta_{CG}=0.1$.

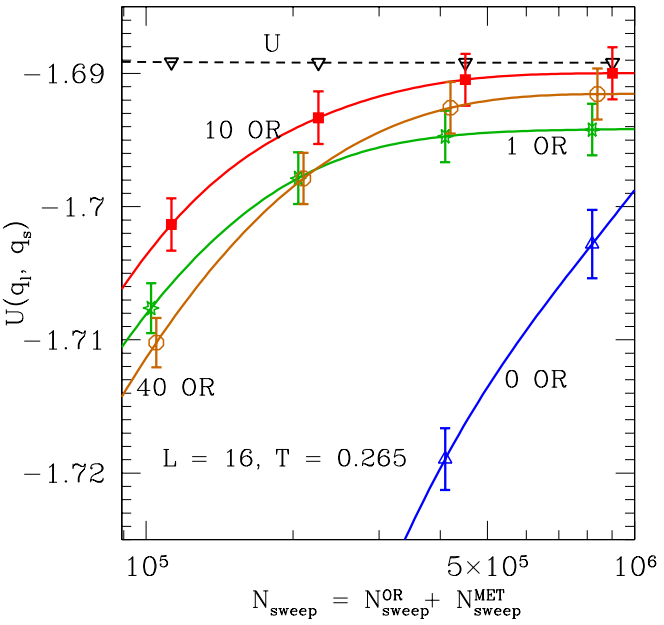

FIG. 9. (Color online) Results for $L=16$, $T=0.265$. The data connected by solid lines is $U(q_l,q_s)$ in Eq. (11) for different number of over-relaxation (OR) sweeps per Metropolis (MET) sweep as indicated. The horizontal axis is the total number of OR plus MET sweeps. The data connected by the dashed line is the energy U, which should equal $U(q_l,q_s)$ in equilibrium according to Eq. (10). Since the energy equilibrates relatively fast, its value does not depend significantly on the ratio of OR to MET sweeps for the range of sweeps presented. The number of parallel tempering sweeps is the same for all sets of data except for "40 OR," where it is 1/4 as many.

We also present data for the spin-glass and chiral-glass susceptibilities in Figs. 7 and 8, respectively. Dividing by $L^{2-\eta}$, where η is a critical exponent, the data should intersect at the critical temperature, see Eq. (13), where η_{SG} is not necessarily equal to η_{CG}. In order to get intersections for $T \simeq 0.30$, where the correlation data merge/intersect for the largest sizes, we took $\eta_{SG}=-0.2$ and $\eta_{CG}=0.1$ in the plots.

Given the large corrections to scaling clearly visible in the data for the correlation lengths, it does not appear possible to get *reliable* estimate of the critical exponents, η_{SG} and η_{CG}, or of the correlation-length exponent ν.

IV. OPTIMIZING THE FRACTION OF OVER-RELAXATION SWEEPS

As already noted, adding over-relaxation steps has been observed[13,14,16] to speed up equilibration. Here we look systematically at how the *ratio* of the number of over-relaxation (OR) sweeps to Metropolis (MET) sweeps alters the *total* number of sweeps needed to equilibrate. In Fig. 9, we plot both sides of Eq. (10), which are equal in equilibrium, for different ratios of the number of OR sweeps to MET sweeps. The data are for $L=16$, $T=0.265$. It is seen that equilibration is considerably sped up by including OR sweeps. It seems that doing 10 OR per MET (which was used in the results in the earlier sections) is somewhat better than 1 OR or 40 OR. Reference 13 argues that of order L, OR sweeps should be done for each MET sweep "to let the microcanonical wave run over the system." Our data are consistent with this, although it seems that the time to equilibrate is not very sensitive to the precise ratio of OR to MET sweeps.

We should emphasize that including OR sweeps not only reduces the number of sweeps to equilibrate, as seen in Fig. 9, but also reduces the CPU time by an even bigger factor because each OR sweep runs several times faster on the computer than an MET sweep.

V. CONCLUSIONS

We have studied the XY spin glass in three dimensions by Monte Carlo simulations using larger sizes than before. We find that the lower critical dimension is close to three. We also find that the behavior of the spin-glass and chiral-glass correlation lengths is strikingly similar, see Fig. 5 and, in our view, does not support the spin-chirality decoupling scenario at least for sizes up to $L=24$.

In earlier work, Maucourt and Grempel[7] have studied the 3D XY spin glass using the domain-wall renormalization group (DWRG) for sizes up to $L=8$. They argue that there is a positive stiffness for the chiralities and, hence a finite-temperature transition, while for spin-glass ordering the system is close to its lower critical dimension. The conclusion for chiralities is different from ours but we note that our sizes are much larger ($L \leq 24$), and that we only see marginal behavior in the chiralities for $L>12$. Furthermore, our approach gives *directly* the correlation lengths, whereas for the DWRG, ground-state energies with different boundary conditions are computed, from which stiffness is *inferred*.

Kawamura and Li[6] used Monte Carlo simulations with sizes up to $L=16$ to compute the overlap function of the

spins and chiralities. In particular, they compute the "Binder ratio," which, like the ratio of the correlation length to system size studied here, is dimensionless. The spin-glass Binder ratio is found to monotonically decrease with increasing L at each temperature. However, we feel that use of the Binder ratio can be tricky near the lower critical dimension, especially when the number of components of the order parameter is high. Since the spin-glass order parameter is quadratic in the spins and the spins have two components, the order parameter has four-components here. The Binder ratio looks at the change in *shape* of the distribution of the (square root of the) order parameter squared summed over all components, when going below the transition. Because of the central limit theorem, there would be *no* change in shape for an infinite number of components. If the number is large the change in shape is small and can easily be masked by corrections to scaling, especially if the system is close to the lower critical dimension, where corrections only fall off very slowly with system size. The use of the Binder ratio for vector spin glasses has also been criticized by Shirakura and Matsubara[21] (they considered explicitly the Heisenberg case). For the chiral-glass Binder ratio, Kawamura and Li estimate a transition temperature from a dip in the data. However, even if the transition is of an unconventional kind (as they claim in order to explain the dip), it seems to us that the Binder ratio should still increase with increasing L at low temperature if there is chiral-glass order. However, this is not observed. We therefore argue that our results, which compute *directly* the relevant correlation lengths, indicate that spin-chirality decoupling does not seem to occur, at least for sizes up to $L=24$. Finally, we find that equilibration is considerably sped up by performing several (perhaps of order L) over-relaxation sweeps per Metropolis sweep (see Fig. 9).

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*peter@physics.ucsc.edu; http://physics.ucsc.edu/~peter

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